

Math 110, Section 101, Quiz 6  
Wednesday, October 4, 2017

This quiz will be graded out of 15 points; the True/False question is worth 3 points, and the exercise is worth 12 points. Please read the instructions carefully, and explain your work.

**True or False.** Mark the following statements as either true or false, or leave a blank if you don't know. A correct answer is worth +1 point, a blank is worth 0 points, and an incorrect answer is worth -1 points, so be smart about guessing!

- a. \_\_\_\_\_ A linear system of equations over a field  $F$  may have exactly two solutions.
- b. \_\_\_\_\_ If the coefficient matrix of a linear system of equations is invertible, then the system has a unique solution.
- c. \_\_\_\_\_ The sum of any two  $n \times n$  elementary matrices is an elementary matrix.

**Solution.** T T F



**Exercise.** Find all solutions to the linear system of equations

$$\begin{aligned}x_1 + x_2 - x_3 &= 1 \\4x_1 + x_2 - 2x_3 &= 2\end{aligned}$$

**Solution.** We first represent the system as an augmented matrix and row reduce to reduced echelon form.

$$\begin{pmatrix} 1 & 1 & -1 & 1 \\ 4 & 1 & -2 & 2 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & -1 & 1 \\ 0 & -3 & 2 & -2 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & -1/3 & 1/3 \\ 0 & 1 & -2/3 & 2/3 \end{pmatrix}$$

The system of equations corresponding to this augmented matrix has the same solution set as the original, and the solutions can be easily read from the new form:

$$\begin{aligned}x_1 + (-1/3)x_3 &= 1/3 \\x_2 + (-2/3)x_3 &= 2/3\end{aligned}$$

Thus the solutions to the system are given by  $x_1 = (1/3)x_3 + 1/3$  and  $x_2 = (2/3)x_3 + 2/3$ , for arbitrary values of  $x_3$ .