Math 110, Section 105, Quiz 5
Wednesday, September 27, 2017

This quiz will be graded out of 15 points; the True/False question is worth 3 points, and the exercise is worth 12 points. Please read the instructions carefully, and explain your work.

True or False. Mark the following statements as either true or false, or leave a blank if you don't know. A correct answer is worth +1 point, a blank is worth 0 points, and an incorrect answer is worth -1 points, so be smart about guessing!

a. ________ For any two finite bases β and β' of a vector space V, there is a change of basis matrix Q such that $Q[x]_\beta = [x]_{\beta'}$ for each $x \in V$.

b. ________ Any finite dimensional vector space V over a field F is isomorphic to some space of matrices $M_{m\times n}(F)$.

c. ________ If S and T are linear transformations such that ST is invertible, then S and T are invertible and $(ST)^{-1} = T^{-1}S^{-1}$

Solution. T T F

✷

Exercise. Let $V \leq P_3(\mathbb{R})$ be the subspace of polynomials of degree at most three given by

$$V = \{p \in P_3(\mathbb{R}) : p(0) = 0\}$$

Let $T : V \to P_2(\mathbb{R})$ be the derivative map, $T : p \mapsto p'$, which we know is a linear transformation. Determine, with proof, whether T is an isomorphism.

Solution. We will show that T is an isomorphism. First, notice that for a polynomial p, we have $p(0) = 0$ iff p has zero constant term. Thus

$$V = \{ax^3 + bx^2 + cx : a, b, c \in \mathbb{R}\}$$

In particular, this is the span of the linearly independent vectors x, x^2, x^3, which thus form a basis β of V. Further, we have

$$T(x) = 1, \quad T(x^2) = 2x, \quad T(x^3) = 3x^2$$

so we see that the image of β under T is likewise a basis of $P_2(\mathbb{R})$. This implies that T is an isomorphism between the two spaces.