Math 110, Section 103, Quiz 4
Wednesday, September 20, 2017

This quiz will be graded out of 15 points; the True/False question is worth 3 points, and the exercise is worth 12 points. Please read the instructions carefully, and explain your work.

True or False. Mark the following statements as either true or false, or leave a blank if you don't know. A correct answer is worth +1 point, a blank is worth 0 points, and an incorrect answer is worth -1 points, so be smart about guessing!

a. ________ If $T, U : V \to V$ are linear transformations on a finite dimensional vector space V, and if β and γ are ordered bases of V, then $[TU]_\beta^\gamma = [UT]_\gamma^\beta$.

b. ________ For a square matrix A, if $A^2 = 0$, then $A = 0$, where here 0 denotes the zero matrix.

c. ________ Matrix multiplication is associative.

Solution. F F T

✷

Exercise. Let $\beta = \{1, x-1, x^2-2x+1\}$ be an ordered basis of the polynomial space $P_2(\mathbb{R})$, and let $S, T : P_2(\mathbb{R}) \to P_2(\mathbb{R})$ be linear transformations with corresponding matrices

$$[S]_\beta = \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 1 & 1 & 1 \end{pmatrix}, \quad [T]_\beta = \begin{pmatrix} 2 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

Compute the polynomial $S(T(x))$.

Solution. Applying a linear transformation L to a vector v in an abstract vector space is equivalent to multiplying the corresopnding column vector by the corresponding matrix represented in a common ordered basis. In particular, if we write x as

$$x = 1 \cdot (1) + 1 \cdot (x-1) + 0 \cdot (x^2 - 2x + 1)$$

then we have that

$$[S(T(x))]_\beta = [S]_\beta [T]_\beta [x]_\beta = \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 2 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

Thus we can compute

$$\begin{pmatrix} 2 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix}, \qquad \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ 4 \end{pmatrix}$$

We conclude that $S(T(x)) = 2 \cdot (1) + 2 \cdot (x-1) + 4 \cdot (x^2 - 2x + 1) = 4x^2 - 6x + 4$.