Math 110, Section 101, Quiz 4
Wednesday, September 20, 2017

This quiz will be graded out of 15 points; the True/False question is worth 3 points, and the exercise is worth 12 points. Please read the instructions carefully, and explain your work.

True or False. Mark the following statements as either true or false, or leave a blank if you don't know. A correct answer is worth +1 point, a blank is worth 0 points, and an incorrect answer is worth -1 points, so be smart about guessing!

a. ________ If V is a finite dimensional vector space and α is an ordered basis of V, then $[I_V]_\alpha = I$.

b. ________ If $T, U : V \to V$ are linear transformations on a finite dimensional vector space V, and if β and γ are ordered bases of V, then $[U+T]_\beta^\gamma = [U]_\beta^\gamma + [T]_\beta^\gamma$.

c. ________ If $A \in M_{n\times p}(\mathbb{R})$ and $B \in M_{m\times n}(\mathbb{R})$, then $AB \in M_{m\times p}(\mathbb{R})$.

Solution. T T F

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Exercise. Let $T : P_2(\mathbb{R}) \to \mathbb{R}^3$ denote the linear transformation given by

$$T : p \mapsto (f'(1), f(0), f''(-1))$$

If $\beta = \{1, x, x^2\}$ is the standard basis of $P_2(\mathbb{R})$ and γ is the standard basis of $\mathbb{R}^3$, compute $[T]_\beta^\gamma$.

Solution. The images of the polynomials in β written in terms of γ are given by

$$\begin{aligned} T(1) &= (0,1,0) = 0\cdot e_1 + 1\cdot e_2 + 0\cdot e_3 \\ T(x) &= (1,0,0) = 1\cdot e_1 + 0\cdot e_2 + 0\cdot e_3 \\ T(x^2) &= (2,0,2) = 2\cdot e_1 + 0\cdot e_2 + 2\cdot e_3 \end{aligned}$$

The matrix $[T]_\beta^\gamma$ is defined to have columns given by the coefficients of the images of the vectors in β, written in terms of the vectors in γ, so the desired matrix is

$$[T]_\beta^\gamma = \begin{pmatrix} 0 & 1 & 2 \\ 1 & 0 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$