Math 110, Section 103, Quiz 3
Wednesday, September 13, 2017

This quiz will be graded out of 15 points; the True/False question is worth 3 points, and the exercise is worth 12 points. Please read the instructions carefully, and explain your work.

True or False. Mark the following statements as either true or false, or leave a blank if you don't know. A correct answer is worth +1 point, a blank is worth 0 points, and an incorrect answer is worth -1 points, so be smart about guessing!

a. ________ The polynomials x^2+x, $3x^2+2x+1$, $x+2$, and x^2-3x-1 are linearly independent in $P_2(\mathbb{R})$.

b. ________ If V is a vector space and $T: V \to V$ is a linear transformation, then T is one-to-one if and only if T is onto.

c. ________ If $x, y, z \in \mathbb{R}^3$ form a basis, then $x+y$, $x+2y$, and $z+y$ also form a basis.

Solution. F F T

✷

Exercise. Find a basis (and show it is a basis) for the set of solutions in $\mathbb{R}^4$ of the linear system of equations

$$\begin{cases} x+y-z-w=0 \\ x-y+z-w=0 \end{cases}$$

Solution. Adding the two equations gives the relation $2x-2w=0$, and subtracting them gives the relation $2y-2z=0$. Thus the equation is solved by all vectors $(x,y,z,w) \in \mathbb{R}^4$ satisfying $x=w$ and $y=z$, i.e. those in the space W, where

$$W = \{(a,b,b,a) \in \mathbb{R}^4 : a, b \in \mathbb{R}\}$$

From the form of the above, we can see that the set $\beta = \{v_1, v_2\}$ with $v_1 = (1,0,0,1)$ and $v_2 = (0,1,1,0)$ gives a basis. For linear independence, note that if $av_1 + bv_2 = 0$, then $(a,b,b,a) = (0,0,0,0)$, so that both a and b must be zero. For spanning, note that any vector of the form $(a,b,b,a) \in W$ can be written as $av_1 + bv_2$. This shows that β is a basis of W.