Math 110, Section 101, Quiz 3
Wednesday, September 13, 2017

This quiz will be graded out of 15 points; the True/False question is worth 3 points, and the exercise is worth 12 points. Please read the instructions carefully, and explain your work.

True or False. Mark the following statements as either true or false, or leave a blank if you don't know. A correct answer is worth +1 point, a blank is worth 0 points, and an incorrect answer is worth -1 points, so be smart about guessing!

a. ________ In $P(\mathbb{R})$, the function $T : p \mapsto p' + x$ is linear, where p' denotes the derivative of p.

b. ________ The set $\{(1, 2, 3), (2, 1, -2), (0, 1, 1), (3, 4, 0)\}$ is linearly independent in $\mathbb{R}^3$.

c. ________ If $T : V \to W$ is a linear transformation and β is a basis of V, then T is determined by its values on β.

Solution. F F T

✷

Exercise. In $V = P_2(\mathbb{R})$, show that the map $T : V \to V$ given by

$$T(p) = x \cdot p'$$

is a linear transformation. Find a basis for $\ker T$ (and show it is a basis).

Solution. To check that T is linear, we need to check that it respects vector space addition and scalar multiplication. In particular, for $p, q \in P_2(\mathbb{R})$, and for $a \in \mathbb{R}$, we have

$$T(p + q) = x \cdot (p + q)' = x \cdot (p' + q') = x \cdot p' + x \cdot q' = T(p) + T(q)$$

and

$$T(ap) = x \cdot (ap)' = x \cdot a(p') = a(x \cdot p') = aT(p)$$

Thus T is linear. To compute $\ker T$, note that for an arbitrary polynomial $p(x) = ax^2 + bx + c$, we have that $T(p) = x \cdot (2ax + b) = 2ax^2 + bx$, which is zero if and only if $a = b = 0$. Thus $\ker T$ consists of the constant polynomials $\{c : c \in \mathbb{R}\}$.

In particular, the set containing only the constant polynomial 1 is a basis of $\ker T$. It spans because for any $c \in \mathbb{R}$, the constant polynomial c can be written as $c \cdot 1$, and thus is in the span of $\{1\}$. Likewise, any single nonzero vector is linearly independent, so we see that $\{1\}$ forms a basis of $\ker T$.