Math 110, Section 103, Quiz 2
Wednesday, September 6, 2017

This quiz will be graded out of 15 points; the True/False question is worth 3 points, and the exercise is worth 12 points. Please read the instructions carefully, and explain your work.

True or False. Mark the following statements as either true or false, or leave a blank if you don't know. A correct answer is worth +1 point, a blank is worth 0 points, and an incorrect answer is worth -1 points, so be smart about guessing!

a. ________ If S is a subset of a vector space V, then $\text{span}(S)$ equals the intersection of all subspaces of V that contain S.

b. ________ Any union of subspaces of a vector space V is a subspace of V.

c. ________ The difficulty of quizzes in your section relative to other sections for the course will not be taken into account at the end of the semester, so it is to your advantage to switch into a section with the easiest quizzes possible.

Solution. T F F

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Exercise. Find a linear dependence relation between the three polynomials in $P(\mathbb{R})$:

$$3x^3 - x^2 + x + 1, \quad x^3 + x^2 + x - 2, \quad -x^3 - 5x^2 - 3x + 9$$

Solution. To find a linear dependence relation, we form an augmented matrix and apply elementary row operations to solve for the necessary coefficients of the relation. For ease of computation, we place the second polynomial in the first column, and the first polynomial in the second column.

$$\begin{pmatrix} 1 & 3 & -1 & 0 \\ 1 & -1 & -5 & 0 \\ 1 & 1 & -3 & 0 \\ -2 & 1 & 9 & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & 3 & -1 & 0 \\ 0 & -4 & -4 & 0 \\ 0 & -2 & -2 & 0 \\ 0 & 7 & 7 & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & 3 & -1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & -4 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

If c_i is the constant associated with column i, we find the solution associated to a choice of $c_3 = 1$ to be

$$(c_1, c_2, c_3) = (4, -1, 1)$$

Thus (keeping in mind that we switched the polynomials corresponding to the first and second columns), the linear dependence relation is given by

$$-1 \cdot (3x^3 - x^2 + x + 1) + 4 \cdot (x^3 + x^2 + x - 2) + 1 \cdot (-x^3 - 5x^2 - 3x + 9) = 0$$