Math 110, Section 105, Quiz 1
Wednesday, August 30, 2017

This quiz will be graded out of 15 points; the True/False question is worth 3 points, and the exercise is worth 12 points. Please read the instructions carefully, and explain your work.

True or False. Mark the following statements as either true or false, or leave a blank if you don't know. A correct answer is worth +1 point, a blank is worth 0 points, and an incorrect answer is worth -1 points, so be smart about guessing!

a. ________ It is an axiom (assumed property) of a vector space that the zero vector of a vector space is unique

b. ________ The set of real-valued functions on $\mathbb{R}$ which are twice-differentiable forms a vector space over $\mathbb{R}$ using the standard addition and scalar multiplication for functions.

c. ________ *Any* two vectors x and y in $\mathbb{R}^2$ are called parallel if $y = tx$ for some nonzero real number t.

Solution. F, T, F.

✷

Exercise. For a fixed integer n, prove that the sum of two polynomials with degree at most n is also a polynomial with degree at most n.

Solution. If a polynomial p has degree at most n, then the highest degree monomial has exponent at most n, and so can be written as

$$p(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$$

where some of the coefficients a_n may be zero. If q is another such polynomial, written as

$$q(x) = b_n x^n + b_{n-1}x^{n-1} + \cdots + b_1 x + b_0$$

then the polynomial sum of p and q can be written as

$$(p+q)(x) = p(x) + q(x) = (a_n + b_n)x^n + (a_{n-1} + b_{n-1})x^{n-1} + \cdots + (a_1 + b_1)x + (a_0 + b_0)$$

From this expression, we see that the degree of $p + q$ is equal to the largest exponent i such that $(a_i + b_i) \neq 0$, and in particular, this exponent is at most n.